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THE MATHEMATICS TEACHER

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A FEW ALGEBRA METHODS.

BY PHILIP R. DEAN.

Granted certain favorable conditions such as: (1) a thoroughly trained and broad-minded teacher, (2) reasonable-sized classes, and (3) pupils mature enough for the work at hand, two things seem to me highly essential for the best teaching. These are (*a*) well-grounded methods; and (*b*) daily and sympathetic planning of each lesson.

Good teaching is hard work even to the man who loves it intensely. We are before a class for forty-five minutes daily: are we realizing our opportunity if we lose time for them through our own poor administration of the work, or our failure to perceive at once any difficulty that arises and its remedy—or again, if we lack that “entire command” of the subject and fresh familiarity with the day’s topics which insure the respect of the whole class?

It is said among public speakers that it is a long lesson to learn to extemporize well, and that the best preparation is to begin by writing out carefully the first speeches and learning them verbatim; after that fewer and fewer words need be written out ahead, but always the great extempore speech consists of material well thought out beforehand but fused afresh into glowing substance by the inspirational fire of the occasion.

So, too, must it be with musical improvisation. Lowell’s “musing organist” “lets his fingers wander as they list” only

when he has mastered the technique and can depend on these mechanical means to express *sua sponte* his deeper feelings.

Is it not thus in our teaching? We must struggle, by long studied practice, to be able to reproduce, the first year perhaps, some one teaching method, which we have thought out carefully; then similarly other methods, until gradually we may move with confidence, and may come before a class with only the immediate subject matter vividly in mind.

This amounts to training the intuition coupled with a mastery of several recognized methods. An experienced resourceful teacher feels quickly when he is not succeeding. Happy he if, in his preparation, he has anticipated this particular situation and is ready with exactly *the* method needed.

I maintain that, even with large classes, it is well worth while to give several minutes before any recitation devising a method especially to reach some backward pupil without sacrificing the others. There is satisfaction in success of this kind; and indeed I have no sympathy with the teacher who says that such and such a pupil "cannot learn mathematics." That teacher is but admitting his own impotence.

George Herbert Palmer gives voice in one of his recent books, to the idea that the great teacher can only be he who delights in *teaching*, and who loves the work for its own sake. We cannot all be great teachers, but we can catch glimpses of the ideal if we can rise to sympathetic touch with the movements of our time and keep our own minds so keenly alive to the learner's view-point that we must ever try to give him our best,—even as *we* should expect from one who stood before *us* to instruct.

I have been in classes under college professors where I was morally certain the instructor had not made a minute's immediate preparation. I got nothing out of it and despised the man. Nay more, I have stood before classes myself when I did not really know, except as I stumbled on them accidentally, what points were the difficult ones to the pupils, nor how to go at these points. I could give the majority of the class perhaps nothing, and, as maybe beads came out on my forehead, I have despised myself in deep humility.

I believe that, with all rigor and exactness, mathematics should be taught psychologically rather than logically. Somewhere, in Preyer or Sully, I read years ago before I was

married, that the young child was certain "vastly to be influenced by his surroundings and in turn vastly to influence them." In my greenness I smiled at this last phrase for I could not see how. And yet as I think of it now, I believe the truest word ever said to teachers is—"And a little child shall lead them."

Methods, then, should not be looked on as a means of saving the teacher time and energy—but rather, each new one should be judged by its possibilities for increasing the teacher's efficiency.

I shall not attempt a summary of well-established standard methods such as the "chalk and talk" of Professor Young, and the methods of awakening interest and checking results discussed by Professor D. E. Smith. I can only mention a few not written so much about.

A good friend of mine, once in an important oral examination, was asked when the "concert method" should be used. He thought a minute and answered, "When the exercise is a written one!" Now this expressed exactly what we were trying, under Mr. Anthony, of DeWitt Clinton High School, six years ago, to do to get at every pupil in our crowded classes. The method was used at various stages of the work. As many pupils as possible were sent to the board; the others at their seats with paper. Dictation (all listening): "The sum of the cubes of a and b ": a second's pause, a signal and all turn and write it quickly. Again: "All write x equals tens digit, y equals units digit:—Attention! Express in algebra the number." At signal all write it. "Attention! Express the number formed by doubling tens digit." Signal after pause and all write. "Attention! Write the algebraic statement that the number formed by doubling tens digit is 200 less than the square of the given number." Pause and then all write. If the questions are "thought-provoking" and the pauses properly timed any one in the room can see at a glance which pupils are doing the thinking and obtaining the answer unaided. Moreover, the teacher has ever before him evidence showing exactly how well the class, as a whole, is keeping up with the development of the topic, as he outlines it, and where drill is needed. We found this method especially good at introducing new branches of the work, and in drilling on algebraic symbolism or shorthand. It demands of the teacher, in a high degree, skill and keenness. He is put in a strategic position where

success or failure depends largely on his own judgment. The best results are reached, as hinted, only after long practice.

Another good time-saving method is to have some one boy bring to the teacher each morning, made out on cards, the assigned examples for the day. The teacher can then mix in a few of his own, including one or two especially designed for the brilliant pupil's wings, and perhaps several review ones, and then be able to have a dozen or twenty pupils at the board working almost as soon as the bell has ceased ringing. This card device also enables the teacher easily to lay aside at the end of the period work which needs consideration the next day. It seems wise to have each pupil put at the top of the board exactly what is on his card, that all may fully understand the problem. After a few minutes of board work the rest of the period may well be given to explaining—all sharing as the teacher directs the questions. Occasionally for review it is well to let each pupil erase and take another card on the same or a different topic as soon as he finishes his example correctly. In this way each pupil may do several during one period. A simple library bureau index scheme keeps the old cards classified, and if the teacher extends the system he may work out a color idea, as blue cards for elementary algebra, yellow for plane geometry, red for trigonometry, etc.

Perhaps I may mention two methods for particular topics. It seems to me that the fundamental theorem of fractions is $a/b = ma/mb$, *i. e.*, that multiplying or dividing both terms of any fraction by the same quantity does not alter its value. If at the beginning of fractions in first-year algebra the pupils are convinced of the truth of this—or in the third-year algebra it is proved outright—the whole subject of adding and subtracting fractions becomes very simple. So, too, most complex

fractions yield readily; *e. g.*,
$$5 + \frac{2}{x} + \frac{3}{x^2}$$
 becomes at once
$$6 - \frac{2}{x^3}$$

$\frac{5x^3 - 2x^2 + 3x}{6x^3 - 2}$. Also the tricky case where one denominator

has a (-1) factor becomes play;—*e. g.*, $\frac{5}{a-b} + \frac{2}{a+b}$

$+\frac{6a}{b^2-a^2}$. The last term of course is seen at once to be
 $-\frac{6a}{a^2-b^2}$. This one theorem so permeates as a principle the

whole subject of fractions that it might well be established at the outset and quoted and used thereafter whenever possible.

"In the spring a young man's fancy lightly turns to thoughts of base ball." Now base-balls mostly move in one curve—the parabola; and I think we owe it to our pupils at least to introduce them to this curve. Most will never get it in any other way. This brings me to my other suggestion. When teaching graphs, is it not worth while—at least in advanced algebra—where we do teach the simplest derivatives now (as in multiple roots and possibly Sturm's functions), to explain the meaning of dy/dx , as applied to a curve, and show how, knowing it, really helps get the direction of the curve at any point and is especially useful in revealing the turning points. I found one class was stimulated to intense interest by a very simple illustrative example in maxima and minima. The teacher has an unusual chance here to suggest the possibilities in higher mathematics and perhaps to interest boys going on. I have found particular success in attempting this work after final examination week. Many will delight in exploring graphs of higher degree—or even transcendental ones—and occasionally really ingenious discoveries will come out from the young Descarteses, Newtons, or Argands and even the girls enthuse over a goodly mixture of colored crayon or the study of families of curves.

One last word may not belong in that inquiry allotted to this paper, but I venture to speak it as it only goes back a step in the study of good teaching. Most of us have something to say as to what grades of work we shall teach during a given term. Many have the planning of a whole department of teachers. Now I ask, if we wish to raise our educational work to the standard of a profession is not the topic elementary mathematics a narrow enough one for one individual in which to be a specialist? We perhaps cannot expect to remember the Greek, or the history which college gave us; but ought not each teacher of mathematics to be at any time as familiar with any topic taught in high school mathematics as he would be if he had taught it himself, say within three years?

It seems to me that if we all could do this, a closer sympathy would grow up in studying mathematical teaching problems and undoubtedly better teaching would result. I would myself like to learn integral calculus through teaching it instead of reviewing it occasionally in summer schools—I would even have the college and technical school professors try a term with some of our beginners. Perhaps the idea of exchanging professors may develop so that something like this may in time be possible. But for the present we secondary men have within our grasp the opportunity to learn well the adolescent mind and the branches of mathematical thought which are deemed suited to it.

I speak feelingly for I myself have made the mistake of teaching no pupils fresh from the elementary schools for several years. When discussion arose as to whether some particular examination question was a good one for the 1a pupil, I found I could not tell *myself* I really knew. This spring, however, I have an advanced algebra class and a beginners' one in successive periods, and I am frank to state that although the classes are exactly the same size—27 in each—the preparation for the latter and the checking of the work require twice as much energy and time from me as does the advanced algebra class.

Possibly a good teacher will do only fair work the first time he teaches a given grade; he ought to do first class work his second time and finished work the third. It seems to me that about four or five repetitions of a grade is all the ordinary teacher can stand together without getting in a rut. Is it not far better to swing around the whole circle occasionally, holding on not by one grade alone but in two or three places at a time? It is easier far to teach only a little field, and I know many teachers who insist they prefer to do so: but in the long run I feel that such a practice does teacher and pupil a wrong—let no man despise thee! Not even the lordly member of the senior class.

To sum up, then; it seems to me (1) In addition to general culture and breadth which we get by keeping alive on current movements and the deeper branches of our work, we need sympathetic touch with our pupils. This is obtained best by spending daily a little time beforehand over what we are asking them to study. (2) Devices which economize time and reach every

pupil in large classes are necessary to-day. Every pupil has a right to an opportunity and we must recognize that some get their earliest perception through one sense and some through another. Good devices often require long practice and great skill from the teacher. (3) Every teacher should, so far as possible, master a grade and the peculiar pedagogic difficulties the first time he teaches it; and should never let himself grow unduly rusty on any part of high school mathematics.

Good *teaching* is what our secondary schools to-day need rather than highly specialized scholarship or philosophical quibbling. Thus will the standard of our calling tend to rise to its true position—the noblest of the professions.

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MATHEMATICS FOR TRAINING AND CULTURE.*

By W. H. METZLER.

To start a reform requires sufficient energy to overcome the inertia of long habit and custom and to get out of the ruts it is often thought necessary to pull with so much force that the wheels are far beyond the level track before the energy is expended or the fact realized.

The history of the teaching of mathematics shows a series of fluctuations from one point of emphasis to another, and all reform carries with it a good deal which in time is set aside as worthless. Some reformers see little or no good in anything of the present, but think because some things are not what they should be all should be upset and an entirely new structure built upon new foundations. True reforms seldom, if ever, come in this way. Certainly the reform in the teaching of mathematics will not, but much of the present and of the past will be found best for the future.

The spirit of the age and country affects much the spirit of the schools, so we find many thinking that the spirit of commercialism, so prevalent in this age and in this country, should guide our school life and teaching. This is the age of commercial and trade schools and I do not think we have enough of

* Read before the New York Section.